Exercise 223

The average temperature (in degrees Celsius) of a city in the northern United States can be modeled by the function $T(x) = 5 + 18\sin\left[\frac{\pi}{6}(x - 4.6)\right]$, where x is time in months and $x = 1.00$ corresponds to January 1. Determine the month and day when the average temperature is 21 °C.

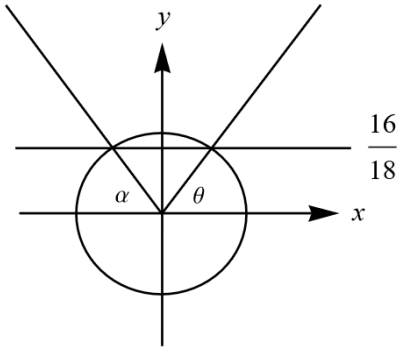
Solution

Set $T(x) = 21$ and solve the equation for x.

$$21 = 5 + 18\sin\left[\frac{\pi}{6}(x - 4.6)\right]$$

$$16 = 18\sin\left[\frac{\pi}{6}(x - 4.6)\right]$$

$$\frac{16}{18} = \sin\left[\frac{\pi}{6}(x - 4.6)\right]$$



There are two angles, α and θ, that give 16/18 after taking the sine. Taking the arcsine of 16/18 on the calculator gives θ. α is $\pi - \theta$.

$$\alpha = \pi - \sin^{-1}\left(\frac{16}{18}\right) = \frac{\pi}{6}(x - 4.6) \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{16}{18}\right) = \frac{\pi}{6}(x - 4.6)$$

$$\frac{6}{\pi}\left[\pi - \sin^{-1}\left(\frac{16}{18}\right)\right] = x - 4.6 \quad \text{or} \quad \frac{6}{\pi}\sin^{-1}\left(\frac{16}{18}\right) = x - 4.6$$

$$\frac{6}{\pi}\left[\pi - \sin^{-1}\left(\frac{16}{18}\right)\right] + 4.6 = x \quad \text{or} \quad \frac{6}{\pi}\sin^{-1}\left(\frac{16}{18}\right) + 4.6 = x$$

$$8.51 \approx x \quad \text{or} \quad 6.69 \approx x$$

Month 8 in the year is August. $0.51(31) = 15.81$, so the day is 15. August 15 is one date when the average temperature is 21 °C.

Month 6 in the year is June. $0.69(30) = 20.7$, so the day is 20. June 20 is one date when the average temperature is 21 °C.